A Mixed-Integer Linear Programming Formulation for "Group 1" Maritime Inventory Routing Instances

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1 Description and Notation

This document describes a mixed-integer linear programming model for "Group 1" instances available on the MIRPLIB webpage `http://mirplib.scl.gatech.edu/`. The LP and MPS files associated with instances in Group 1 are based on this formulation. These instances correspond to a set of deterministic single product maritime inventory routing problems (MIRPs).

Indices and sets

$t \in \mathcal{T}$	set of time periods with $T = \lvert\mathcal{T}\rvert$
$v \in \mathcal{V}$	set of vessels
$j \in \mathcal{J}^P$	set of production, a.k.a. loading, ports
$j \in \mathcal{J}^C$	set of consumption, a.k.a. discharging, ports
$j \in \mathcal{J}$	set of all ports: $\mathcal{J} = \mathcal{J}^P \cup \mathcal{J}^C$
$n \in \mathcal{N}$	set of regular nodes or port-time pairs: $\mathcal{N} = \{n = (j,t) : j \in \mathcal{J}, t \in \mathcal{T}\}$
$n \in \mathcal{N}_{s,t}$	set of all nodes (including a source node n_s and a sink node n_t)
$a \in \mathcal{A}$	set of all arcs
$a \in \mathcal{A}^v$	set of arcs associated with vessel $v \in \mathcal{V}$
$a \in \mathcal{FS}_n^v$	forward star (set of all outgoing arcs) associated with node $n = (j,t) \in \mathcal{N}_{s,t}$ and vessel $v \in \mathcal{V}$
$a \in \mathcal{RS}_n^v$	reverse star (set of all incoming arcs) associated with node $n = (j,t) \in \mathcal{N}_{s,t}$ and vessel $v \in \mathcal{V}$

Data

$\alpha_{j,t}^{\max}$	bound on the amount of product that can be bought from or sold to the spot market in a single period
$\alpha_j^{\max}$	bound on the cumulative amount of product that can be bought from or sold to the spot market over the entire planning horizon
B_j	number of berths (berth limit) at port $j \in \mathcal{J}$
C_a^v	cost for vessel v to traverse arc $a = ((j_1, t_1), (j_2, t_2)) \in \mathcal{A}^v$
$d_{j,t}$	amount produced/consumed at port $j \in \mathcal{J}$ in period t
Δ_j	an indicator parameter taking value +1 if $j \in \mathcal{J}^P$ and -1 if $j \in \mathcal{J}^C$
ϵ_z	nonnegative cost parameter associated with attempting to load or discharge at a port
$F_{j,t}^{\min}$ ($F_{j,t}^{\max}$)	minimum (maximum) amount of product that can be loaded/discharged at port j from a single vessel in a period
$P_{j,t}$	nonnegative penalty parameter associated with one unit of lost production or stockout at port j in time period t
Q^v	capacity of vessel $v \in \mathcal{V}$ (capacity of a vessel in vessel class vc)
R_n	the unit sales revenue for product discharged at node $n = (j, t)$
$S_{j,t}^{\min}$ ($S_{j,t}^{\max}$)	lower bound (capacity) at port $j \in \mathcal{J}$ in time period $t \in \mathcal{T}$
$s_{j,0}$	initial inventory at port $j \in \mathcal{J}$
s_0^v	initial inventory on vessel $v \in \mathcal{V}$

Decision Variables

$\alpha_{j,t}$	(continuous) amount of product that port j purchases from (when $j \in \mathcal{J}^C$) or sells to (when $j \in \mathcal{J}^P$) the spot market in time period t
f_n^v	(continuous) amount loaded/discharged at port $j \in \mathcal{J}$ in period t from vessel $v \in \mathcal{V}$
$s_{j,t}$	(continuous) number of units of inventory at port $j \in \mathcal{J}$ available at the *end* of period t
s_t^v	(continuous) number of units of inventory on vessel $v \in \mathcal{V}$ available at the *end* of period t
x_a^v	(binary) takes value 1 if vessel $v \in \mathcal{V}$ uses arc a incident to node $n = (j, t) \in \mathcal{N}$
z_n^v	(binary) takes value 1 if vessel $v \in \mathcal{V}$ can load or discharge product at node $n = (j, t) \in \mathcal{N}$

Network

The model takes place on an underlying time-space network. The network has a set $\mathcal{N}_{s,t}$ of nodes and a set $\mathcal{A}$ of directed arcs. The node set is shared by all vessels, while each vessel has its own arc set $\mathcal{A}^v$. The set $\mathcal{N}_{s,t}$ of nodes consists of "regular" nodes or port-time pairs, which represent a potential visit by one or more vessels to port $j \in \mathcal{J}$ in time period $t \in \mathcal{T}$, as well as a source node n_s and a sink node n_t.

Associated with each vessel v is a set $\mathcal{A}^v$ of arcs, which can be subdivided further as shown in Figure 1. An arc from the source to the sink node denotes that the vessel is not used in the solution. A source arc from the source node to a regular node represents the arrival of a vessel to its initial destination. A sink arc from a regular node to the sink node conveys that a vessel is no longer being used and has exited the system. A waiting arc from a port j in time period t to the same port in time period $t + 1$ represents that a vessel stays at the same port in two consecutive time periods. Finally, a travel arc from a regular node $n_1 = (j_1, t_1)$ to a regular node $n_2 = (j_2, t_2)$ with $j_1 \neq j_2$ represents travel between two distinct ports. The set of incoming and outgoing arcs associated with vessel $v \in \mathcal{V}$ at node $n \in \mathcal{N}_{s,t}$ are denoted by $\mathcal{RS}_n^v$ (for

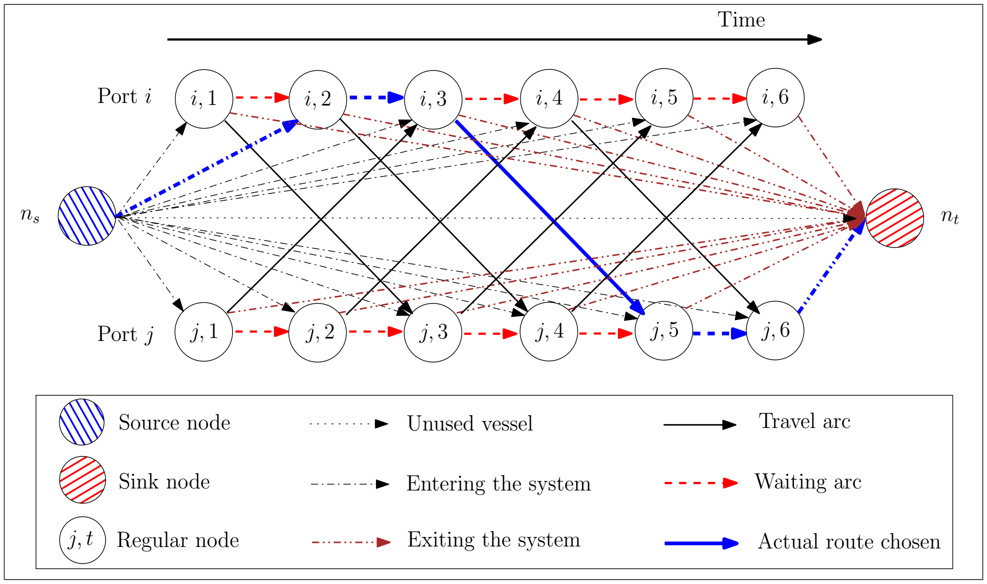

Figure 1: Example of time-space network structure for a single vessel

reverse star) and $\mathcal{FS}_n^v$ (for forward star), respectively.

The network structure affords great flexibility in modeling and embeds a significant amount of data. First, note that the travel duration between two distinct ports on a travel arc is given by the length $(t_2 - t_1)$ of the arc and this duration may be time-dependent, e.g., it may take longer to travel from China to Europe during a particular season. Second, in some applications, all vessels may not be able to visit all ports because of physical restrictions at the port. Such vessel-port incompatibilities can easily be handled in this network by simply not including arcs in the respective sets. For example, if vessel v cannot visit port j, then the sets $\mathcal{FS}_n^v$ and $\mathcal{RS}_n^v$ are empty for all $n = (j, t)$ and $t \in \mathcal{T}$.

2 An Arc-Based Mixed-Integer Linear Programming Model

Below, a discrete-time arc-flow formulation is presented. The "flow" of each individual vessel is model. See [2] for more details.

Group 1 MIP Model

$$\max \quad \sum_{n\in\mathcal{N}}\sum_{v\in\mathcal{V}} R_n f_n^v - \sum_{v\in\mathcal{V}}\sum_{a\in\mathcal{A}^v} C_a^v x_a^v - \sum_{v\in\mathcal{V}}\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}(t\epsilon_z) z_{j,t}^v - \sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}} P_{j,t}\alpha_{j,t} \tag{1a}$$

$$\text{s.t.} \quad \sum_{a\in\mathcal{FS}_n^v} x_a^v - \sum_{a\in\mathcal{RS}_n^v} x_a^v = \begin{cases} +1 & \text{if } n = n_s \\ -1 & \text{if } n = n_t \\ 0 & \text{if } n \in \mathcal{N} \end{cases}, \qquad \forall\, n \in \mathcal{N}_{s,t}, \forall\, v \in \mathcal{V} \tag{1b}$$

$$s_{j,t} = s_{j,t-1} + \Delta_j \left(d_{j,t} - \sum_{v\in\mathcal{V}} f_n^v - \alpha_{j,t} \right), \qquad \forall\, n = (j,t) \in \mathcal{N} \tag{1c}$$

$$s_t^v = s_{t-1}^v + \sum_{\{n=(j,t)\in\mathcal{N}\}} \Delta_j f_n^v, \qquad \forall\, t \in \mathcal{T}, \forall\, v \in \mathcal{V} \tag{1d}$$

$$\sum_{v\in\mathcal{V}} z_n^v \le B_j, \qquad \forall\, n = (j,t) \in \mathcal{N} \tag{1e}$$

$$z_n^v \le \sum_{a\in\mathcal{RS}_n^v} x_a^v, \qquad \forall\, n = (j,t) \in \mathcal{N}, \forall\, v \in \mathcal{V} \tag{1f}$$

$$s_t^v \ge Q^v x_a^v, \qquad \forall\, v \in \mathcal{V}, \forall\, a = ((j_1,t),(j_2,t')) \in \mathcal{A}^v : j_1 \in \mathcal{J}^P, j_2 \in \mathcal{J}^C \cup \{n_t\} \tag{1g}$$

$$s_t^v \le Q^v(1 - x_a^v), \quad \forall\, v \in \mathcal{V}, \forall\, a = ((j_1,t),(j_2,t')) \in \mathcal{A}^v : j_1 \in \mathcal{J}^C, j_2 \in \mathcal{J}^P \cup \{n_t\} \tag{1h}$$

$$\sum_{t\in\mathcal{T}} \alpha_{j,t} \le \alpha_j^{\max} \qquad \forall\, j \in \mathcal{J} \tag{1i}$$

$$0 \le \alpha_{j,t} \le \alpha_{j,t}^{\max} \qquad \forall\, j \in \mathcal{J}, \forall\, t \in \mathcal{T} \tag{1j}$$

$$F_{j,t}^{\min} z_{j,t}^v \le f_{j,t}^v \le F_{j,t}^{\max} z_{j,t}^v, \qquad \forall\, n = (j,t) \in \mathcal{N}, \forall\, v \in \mathcal{V} \tag{1k}$$

$$S_{j,t}^{\min} \le s_{j,t} \le S_{j,t}^{\max}, \qquad \forall\, n = (j,t) \in \mathcal{N} \tag{1l}$$

$$0 \le s_t^v \le Q^v, \qquad \forall\, v \in \mathcal{V}, \forall\, t \in \mathcal{T} \tag{1m}$$

$$x_a^v \in \{0,1\}, \qquad \forall\, v \in \mathcal{V}, \forall\, a \in \mathcal{A}^v \tag{1n}$$

$$z_n^v \in \{0,1\}, \qquad \forall\, n = (j,t) \in \mathcal{N}, \forall\, v \in \mathcal{V}. \tag{1o}$$

Objective function. The objective function is stated in the form of a profit maximization where revenue is earned at the time product is delivered to a port. Inventory costs are not included in the objective function because we assume that the shipper owns both the production and consumption sites.

When a vessel visits a port, there may be multiple time periods in which it can load or discharge product. In reality, we prefer a vessel to load or discharge as few times as possible to minimize the duration and cost of port operations associated with that vessel. In addition, we prefer a vessel to load or discharge as soon as it arrives at a port, assuming the port has a berth available and enough inventory or capacity to do so. To accommodate these secondary goals without affecting the primary goals of managing inventory and routing vessels, we may choose to associate a negligible cost $t\epsilon_z$ with each binary decision variable $z_{j,t}^v$, where ϵ_z is a small nonnegative parameter representing the cost to load or discharge and t is the time period. If a nonzero ϵ_z parameter is specified, the objective function should include the additional term

$$\sum_{v\in\mathcal{V}}\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}} -(t\epsilon_z) z_{j,t}^v .$$

Note that by using the coefficient $-(t\epsilon_z)$ instead of $-\epsilon_z$, solutions in which a vessel attempts to load or discharge sooner rather than later are preferred.

Flow balance constraints for vessels. Constraints (1b) require flow balance for every vessel, that is, if a vessel enters node $n \in \mathcal{N}$, it must also exit node $n \in \mathcal{N}$.

Inventory balance constraints. Constraints (1c) are inventory balance constraints at loading and discharging ports, respectively. Constraints (1d) maintain inventory balance on each vessel.

Constraints related to attempting to load or discharge. Constraints (1e) limit the number of vessels that can attempt to load/discharge at a port at a given time. Constraints (1f) ensure that a vessel does not attempt to load/discharge at a node unless the vessel is actually at that node. Constraints (1k) state that if a vessel attempts to load/discharge at node n, then the actual amount loaded/discharged is within predetermined port-specific bounds $[F_{j,t}^{\min}, F_{j,t}^{\max}]$.

"Travel at capacity" / "Travel empty" constraints. Constraints (1g) and (1h) require a vessel to travel at capacity from a loading region to a discharging region and empty from a discharging region to a loading region. Although these "travel full" constraints (1g) are usually justified on the basis that vessel capacity is a scarce resource and therefore a vessel's capacity should always be fully utilized when making long voyages, there are applications in which it has been shown that such an assumption may not always be optimal (see, e.g., [1]). On the other hand, in virtually all MIRPs discussed in the literature, vessels fully discharge before reloading. This is in contrast to what occurs in liner shipping where vessels load and discharge containers regularly without ever fully discharging. Finally, note that constraints (1g) and (1h) require vessels to leave the system empty or full.

Simplified spot market constraints. In reality, there are often so-called spot markets available where product can be purchased or sold. Since accurately modeling spot market availability and price dynamics is difficult, we assume that a simplified spot market exists so that if a discharging port is on the brink of a stockout, it may purchase product from a nearby spot market to avoid stocking out. Similarly, if a loading port is at risk of reaching capacity, it may sell product to a spot market to avoid having to halt production. On the other hand, spot markets are not always available, so we also assume that there is a limit on the cumulative amount of product that can be purchased from or sold to a spot market from each port. In short, Constraints (1j) bound the amount of product that can be purchased from or sold to a spot market in a single period by a constant $\alpha_{j,t}^{\max}$. Constraints (1i) limit the cumulative amount of product that can be purchased from or sold to a spot market by each port over the entire planning horizon by a constant $\alpha_j^{\max}$.

Variable bounds. Constraints (1l) require ending inventory in each time period at each port to be within prespecified bounds. Constraints (1m) require ending inventory in each time period on each vessel to be within prespecified bounds.

Side notes. It is also worth noting some not-so-obvious features and constraints that are not stated (implicitly or explicitly) in the Formulation (1). First, although the notion of a region is not mentioned, in our library of instances, we assume that each port belongs to a prespecified region of the same type, i.e., loading or discharging. Second, it is assumed that if a vessel travels from port i to port j, the vessel will attempt to load/discharge at port j (and, therefore, incurs a port fee). This will always happen in an optimal solution because the data for the instances of interest all satisfy the triangle inequality, i.e., it is cheaper to travel from port a to port c than to travel from a to b and then b to c. Note that the port fee is paid only once. That is, if a vessel attempts to load at port j in period t, remains at port j in period $t+1$ (but, perhaps, abandons the berth in this period), and then attempts to load again at port j in period $t+2$, only one port fee is incurred. Third, in a single time period, it may be possible for a vessel to load or discharge more inventory than a port's capacity. For example, suppose a discharging port j consumes 25 units of product per period and has a constant capacity of 250 units. Then, 275 units could be discharged in a single period. This could occur if port j has 0 inventory at the end of period t, i.e., $s_{j,t} = 0$, and a vessel carrying at least 275 units of inventory arrives in period $t+1$ and discharges 275 units, 25 of which satisfy demand in period $t+1$ while the remaining 250 units are stored in inventory. This example also shows the limitations of a discrete-time formulation for operational planning since inventory bounds are only required to be satisfied at the end of each period. Fourth, there is no backlogging of inventory. Rather, inventory bounds at ports are satisfied at the end of each time period t.

References

[1] M. Fodstad, K. T. Uggen, F. Rømo, A. Lium, and G. Stremersch. LNGScheduler: A rich model for coordinating vessel routing, inventories and trade in the liquefied natural gas supply chain. *Journal of Energy Markets*, 3(4):31–64, 2010.

[2] D. J. Papageorgiou, M.-S. Cheon, A. B. Keha, G. L. Nemhauser, and J. Sokol. MIRPLib: A maritime inventory routing survey and instance library. *Submitted*, 2012.