A Mixed-Integer Linear Programming Formulation for "Group 2" Maritime Inventory Routing Instances

Dimitri J. Papageorgiou

H. Milton Stewart School of Industrial and Systems Engineering

Georgia Institute of Technology

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1 Description and Notation

This document describes a mixed-integer linear programming model for "Group 2" instances available on the MIRPLIB webpage `http://mirplib.scl.gatech.edu/`. The LP and MPS files associated with instances in Group 2 are based on this formulation. These instances correspond to a set of long-horizon deterministic single product maritime inventory routing problems (MIRPs). Throughout, it is assumed that there is one port per region, so the term "port" and "region" are used synonymously.

Indices and sets

$t \in \mathcal{T}$	set of time periods with $T = \lvert\mathcal{T}\rvert$
$vc \in \mathcal{VC}$	set of vessel classes
$j \in \mathcal{J}^P$ ($r \in \mathcal{R}^P$)	set of production, a.k.a. loading, ports (regions)
$j \in \mathcal{J}^C$ ($r \in \mathcal{R}^C$)	set of consumption, a.k.a. discharging, ports (regions)
$j \in \mathcal{J}$ ($r \in \mathcal{R}$)	set of all ports (regions): $\mathcal{J} = \mathcal{J}^P \cup \mathcal{J}^C$ and $\mathcal{R} = \mathcal{R}^P \cup \mathcal{R}^C$
$n \in \mathcal{N}$	set of regular nodes or port-time pairs: $\mathcal{N} = \{n = (j,t) : j \in \mathcal{J}, t \in \mathcal{T}\}$
$n \in \mathcal{N}_{s,t}$	set of all nodes (including a source node n_s and a sink node n_t)
$\mathcal{A}^{vc}$	set of arcs associated with vessel class $vc \in \mathcal{VC}$
$\mathcal{A}^{vc,\text{inter}}$	set of inter-regional arcs associated with vessel class $vc \in \mathcal{VC}$
$a \in \mathcal{FS}_n^{vc}$	forward star (set of outgoing arcs) associated with node $n = (j,t) \in \mathcal{N}_{s,t}$ and vessel class $vc \in \mathcal{VC}$
$a \in \mathcal{RS}_n^{vc}$	reverse star (set of incoming arcs) associated with node $n = (j,t) \in \mathcal{N}_{s,t}$ and vessel class $vc \in \mathcal{VC}$
$a \in \mathcal{FS}_n^{vc,\text{inter}}$	forward star of inter-regional arcs for node $n = (j,t) \in \mathcal{N}_{s,t}$ and vessel class $vc \in \mathcal{VC}$

Data

B_j (B_r)	number of berths (berth limit) at port $j \in \mathcal{J}$ (in region $r \in \mathcal{R}$)
C_a^{vc}	unit cost for a vessel in vessel class vc to traverse arc $a = ((j_1, t_1), (j_2, t_2)) \in \mathcal{A}^{vc}$
$d_{j,t}$	number of units produced/consumed at port $j \in \mathcal{J}$ in period $t \in \mathcal{T}$
Δ_j (Δ_r)	an indicator parameter taking value +1 if $j \in \mathcal{J}^P$ ($r \in \mathcal{R}^P$) and -1 if $j \in \mathcal{J}^C$ ($r \in \mathcal{R}^C$)
$P_{j,t}$	nonnegative penalty parameter associated with one unit of lost production or stockout at port j in time period t
Q^{vc}	capacity of a vessel in vessel class vc
$S_{j,t}^{\min}$ ($S_{j,t}^{\max}$)	lower bound (capacity) at port $j \in \mathcal{J}$ in time period $t \in \mathcal{T}$
$s_{j,0}$	initial inventory at port $j \in \mathcal{J}$

Decision Variables

$\alpha_{j,t}$	(continuous) amount to produce/consume at port $j \in \mathcal{J}$ in period t
$s_{j,t}$	(continuous) number of units of inventory at port $j \in \mathcal{J}$ available at the *end* of period t
x_a^{vc}	(integer) number of vessels in vessel class vc that take arc a

2 An Arc-Based Mixed-Integer Linear Programming Model

Below, a discrete-time arc-flow formulation is presented. Flows of vessel classes are model; the flow of each individual vessel is not modeled. See [2] for more details concerning MIRPs.

We consider the following MIP model:

Group 2 MIP Model

$$\max \quad \sum_{vc \in \mathcal{VC}} \sum_{a \in \mathcal{A}^{vc}} -C_a^{vc} x_a^{vc} + \sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} -P_{j,t} \alpha_{j,t} \tag{1a}$$

$$\text{s.t.} \quad \sum_{a \in \mathcal{FS}_n^{vc}} x_a^{vc} - \sum_{a \in \mathcal{RS}_n^{vc}} x_a^{vc} = \begin{cases} +1 & \text{if } n = n_s \\ -1 & \text{if } n = n_t \\ 0 & \text{if } n \in \mathcal{N} \end{cases}, \qquad \forall\, n \in \mathcal{N}_{s,t}, \forall\, vc \in \mathcal{VC} \tag{1b}$$

$$s_{j,t} = s_{j,t-1} + \Delta_j \left(d_{j,t} - \sum_{vc \in \mathcal{VC}} \sum_{a \in \mathcal{FS}_n^{vc,\text{inter}}} Q^{vc} x_a^{vc} - \alpha_{j,t} \right), \qquad \forall\, n = (j,t) \in \mathcal{N} \tag{1c}$$

$$\sum_{vc \in \mathcal{VC}} \sum_{a \in \mathcal{FS}_n^{vc,\text{inter}}} x_a^{vc} \le B_j, \qquad \forall\, n = (j,t) \in \mathcal{N} \tag{1d}$$

$$\alpha_{j,t} \ge 0, \qquad \forall\, n = (j,t) \in \mathcal{N} \tag{1e}$$

$$s_{j,t} \in [S_{j,t}^{\min}, S_{j,t}^{\max}], \qquad \forall\, n = (j,t) \in \mathcal{N} \tag{1f}$$

$$x_a^{vc} \in \{0,1\}, \qquad \forall\, vc \in \mathcal{VC}, \forall\, a \in \mathcal{A}^{vc,\text{inter}} \tag{1g}$$

$$x_a^{vc} \in \mathbb{Z}_+, \qquad \forall\, vc \in \mathcal{VC}, \forall\, a \in \mathcal{A}^{vc} \setminus \mathcal{A}^{vc,\text{inter}}. \tag{1h}$$

The objective is to minimize the sum of all transportation costs and penalties for lost production and stockout. Constraints (1b) require flow balance of vessels within each vessel class. Constraints (1c) are inventory balance constraints at loading and discharging ports, respectively. Berth limit constraints (1d) restrict the number of vessels that can attempt to load/discharge at a port at a given time. This formulation requires that a vessel must travel at capacity from a loading region to a discharging region and empty from a discharging region to a loading region. In contrast to other models in the literature, Formulation (1) does not require decision variables for tracking inventory on vessels (vessel classes), nor does it include decisions variables for the quantity loaded/discharged in a given period. Bound constraints (1g) ensure that at most one vessel per vessel class travels along a particular inter-regional arc. In a maritime setting, it is extremely rare for two vessels to begin travel between the same two ports in the same period.

In order for Formulation (1) to furnish the correct lost production and stockout values, the penalty parameters $P_{j,t}$ must be monotonically decreasing in time, i.e., $P_{j,t} > P_{j,t+1}$. This ensures that a solution will not involve lost production (stockout) until the inventory level reaches capacity (falls to zero).

This model is similar to the one studied in Goel et al. [1]. The major differences are that they do not include travel costs in the objective function; they model each vessel individually (in other words, there is only one vessel per vessel class); they model consumption rates as decision variables with upper and lower bounds; and they include an additional set of continuous decision variables to account for cumulative unmet demand at each consumption port.

References

[1] V. Goel, K. C. Furman, J.-H. Song, and A. S. El-Bakry. Large neighborhood search for LNG inventory routing. *Journal of Heuristics*, 18(6):821–848, Dec. 2012.

[2] D. J. Papageorgiou, M.-S. Cheon, A. B. Keha, G. L. Nemhauser, and J. Sokol. MIRPLib: A maritime inventory routing survey and instance library. *Submitted*, 2012.